Lensing Shear Estimators

- Issues and the need for Estimators
- Three Basic Methods
  - KSB - Kaiser, Squires, & Broadhurst 1995
  - RRG - Rhodes, Refregier, & Groth 2000
  - Shapelets - Refregier 2001
- Description & Evaluation

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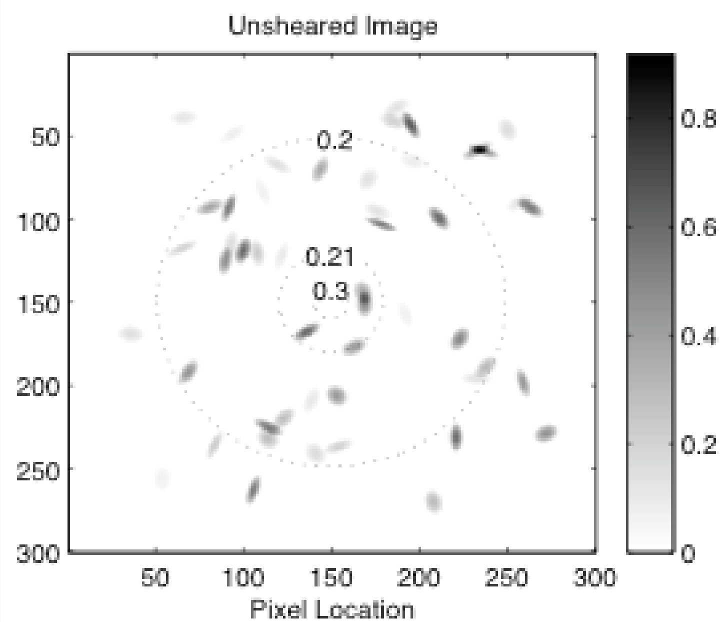
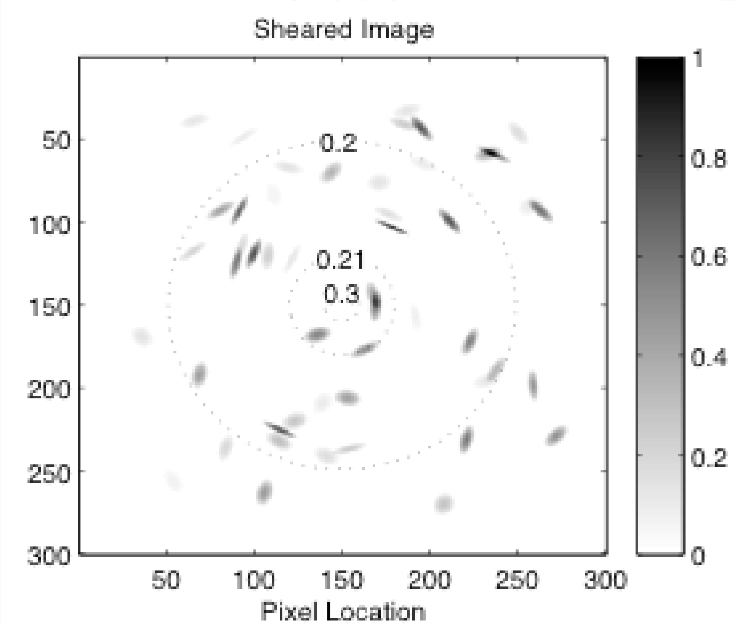
Synopsis of Weak Lensing

- Weak lensing distortions are small
  - ~10% for clusters
  - ~2% for large scale structure
- Galaxies orient themselves in a random nature ⇨ effects of weak lensing must be done on statistical basis
- Weak lensing provides information about the dark matter distribution, thus providing information about the cosmological parameters

In an ideal world…

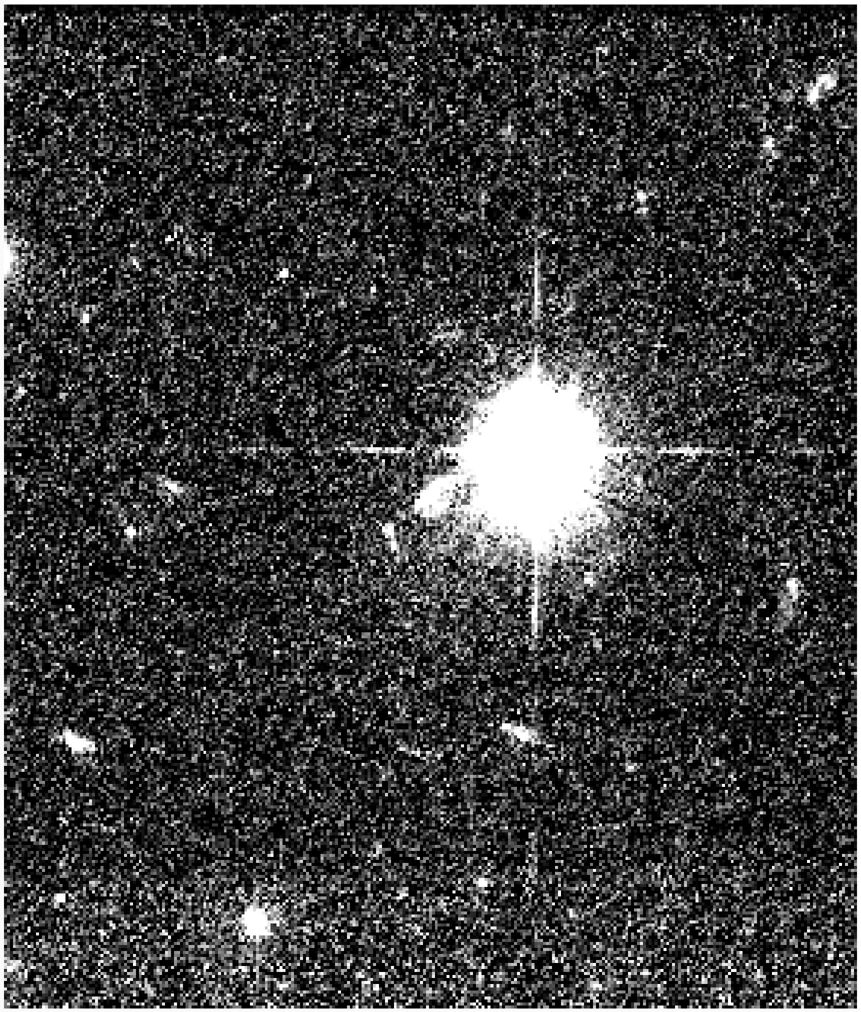




- There would be no PSF, especially not anisotropic
- Cosmic Rays would not strike detectors
- The perfect weight function would always be 1
- Drizzling would not “round” images
- There would be no noise

…Gravitational lenses would produce ideal images and we could use simple shear estimators

In reality…



Goods Data

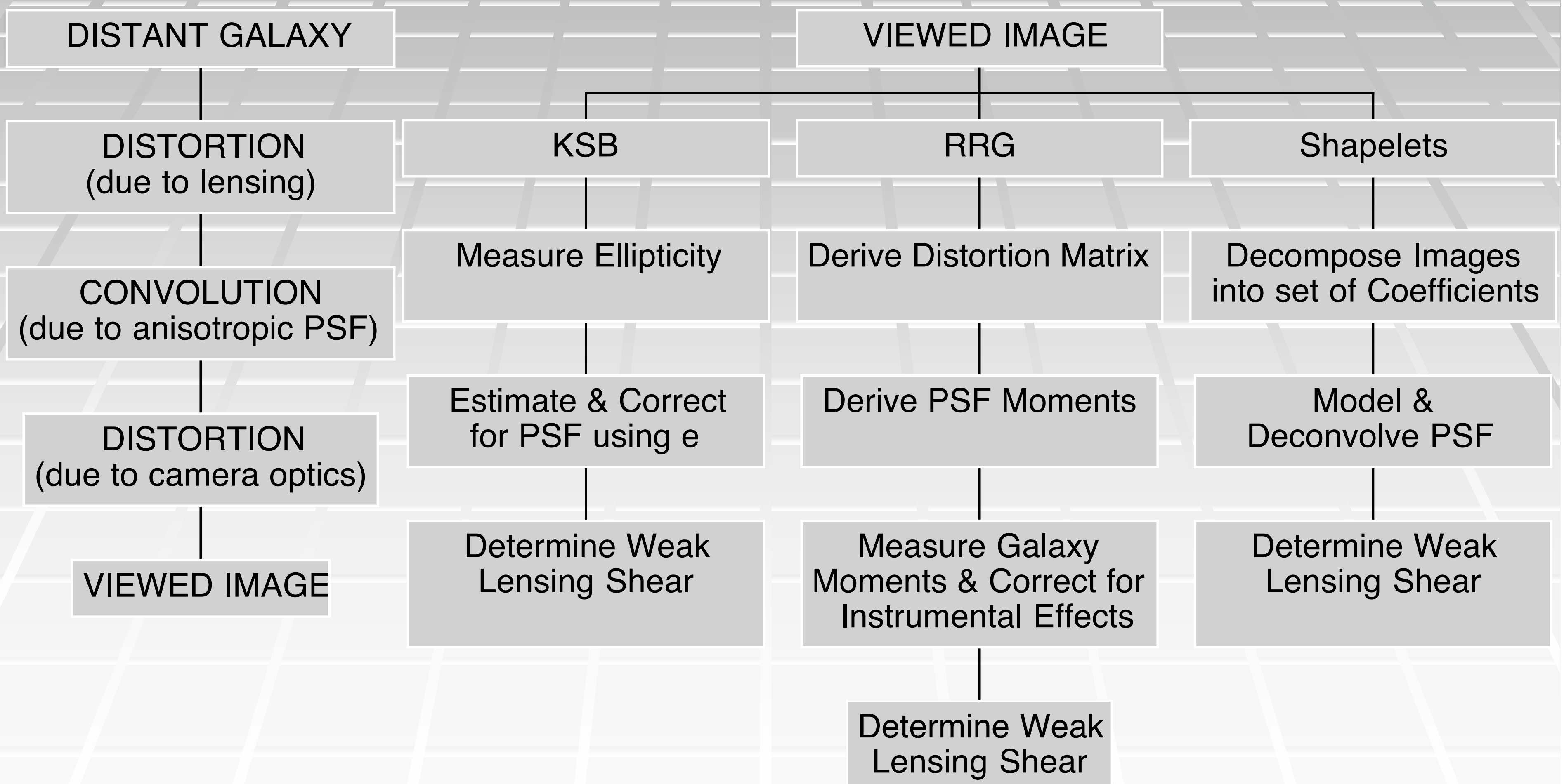
- We have zodical noise, readout noise, Poisson noise
- Reducing PSFs by space-based observations increases cosmic rays
- PSFs are anisotropic
- Drizzling and Dithering results in errors

…images are far from ideal

How do we measure the shear in these polluted images

Polluted Images

What's happening? | How can it be fixed?



KSB

2nd Brightness Moments of the Image:

$$\mathrm{Q}_{ij} = \int d^2\theta \theta_i \theta_j I(\theta) W(\theta)$$

Ellipticity: $\epsilon_1 = \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}}$; $\epsilon_2 = \frac{2Q_{12}}{Q_{11} + Q_{22}}$

$$\epsilon^2 = \epsilon_1^2 + \epsilon_2^2$$

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Shear: $\gamma_i \approx \epsilon_i / 2$

$W(\theta)$ = Weighting Function

θ = Location on Image

$I(\theta)$ = Light at θ

The Mathematical Ease of KSB

- Although KSB *looks* mathematically complicated, it is quite simple:
  1. Choose a weight function
  2. Take the sum of the galaxy, the weight function, and position
  3. Compute the ellipticity and the shear

Let x_0 and y_0 be the center of the galaxy

Unity, Gaussian, and Elliptical Gaussian are possible weight functions

$$W(\theta) = 1$$

$$W(\theta) = \exp\left[\frac{(x-x_0)^2+(y-y_0)^2}{\sigma^2}\right]$$

$$W(\theta) = \exp\left[\frac{((x-x_0)/a)^2-((y-y_0)/b)^2}{\sigma^2}\right]$$

$$Q_{xx} = \sum_{x,y}(\text{galaxy}(x,y) \times W(\theta) \times (x-x_0)^2)$$

$$Q_{xy} = \sum_{x,y}(\text{galaxy}(x,y) \times W(\theta) \times (x-x_0)(y-y_0))$$

$$Q_{yy} = \sum_{x,y}(\text{galaxy}(x,y) \times W(\theta) \times (y-y_0)^2)$$

$$\epsilon_1 = \frac{Q_{xx}-Q_{yy}}{Q_{xx}+Q_{yy}},\ \epsilon_2 = \frac{2Q_{xy}}{Q_{xx}+Q_{yy}}$$

$$\gamma_1 \approx \frac{\epsilon_1}{2},\ \gamma_2 \approx \frac{\epsilon_2}{2}$$

$$\gamma^2 = \gamma_1^2 + \gamma_2^2$$

RRG

Multipole Moments of the Image:

$$\mathrm{I} \equiv \int d^2\theta w(\theta) i(\theta),$$
$$\mathrm{I}_{ij} \equiv \int d^2\theta \theta_i \theta_j w(\theta) i(\theta),$$
$$\mathrm{I}_{ijk} \equiv \int d^2\theta \theta_i \theta_j \theta_k w(\theta) i(\theta), \text{ etc}$$

Normalized: $J_{ij} \equiv I_{ij}/I, J_{ijk} \equiv I_{ijk}/I$, etc.

Ellipticity: $\epsilon_i \equiv \{J_{11} - J_{22}, 2J_{12}\}/(J_{11} + J_{22})$

Shear: $\gamma_i = G^{-1}\langle\epsilon_i\rangle + O(\phi^2)$

$$G \equiv 2 - \langle\epsilon^2\rangle - \tfrac{1}{2}\langle\lambda\rangle - \tfrac{1}{2}\langle\epsilon \cdot \mu\rangle$$

$$\lambda \equiv (J_{1111} + 2J_{1122} + J_{2222})/\tfrac{1}{2}(J_{11} + J_{22})w^2$$

$$\mu_1 = (-J_{1111} + J_{2222})/\tfrac{1}{2}(J_{11} + J_{22})w^2, \ \mu_2 = -2(J_{1112} + J_{1222})/\tfrac{1}{2}(J_{11} + J_{22})w^2$$

Shapelets

Basis Functions:

1-dimension: $\phi_n(x) \equiv \left[2^n \pi^{\frac{1}{2}} n!\right]^{-\frac{1}{2}} H_n(x) e^{-\frac{x^2}{2}}$

2-dimension: $\phi_n(X) \equiv \phi_{n_1}(x_1)\phi_{n_2}(x_2)$

Basis function: $B_n(X;\beta) \equiv \beta^{-1}\phi_n(\beta^{-1}X)$

Coefficients: $f_n = \int d^2x f(X) B_n(X;\beta)$ (Unlensed)

Shear: $\tilde{\gamma}_{in} = \frac{f_n' - \langle f_n \rangle}{S_{inm}\langle f_m \rangle}$, i=1 n_1, n_2 even; i=2 $n_1, n2$ odd

$f(X)$ = Intensity of galaxy

S_{imn} = Shear Generator Matrix

f_n' = Lensed Coefficients

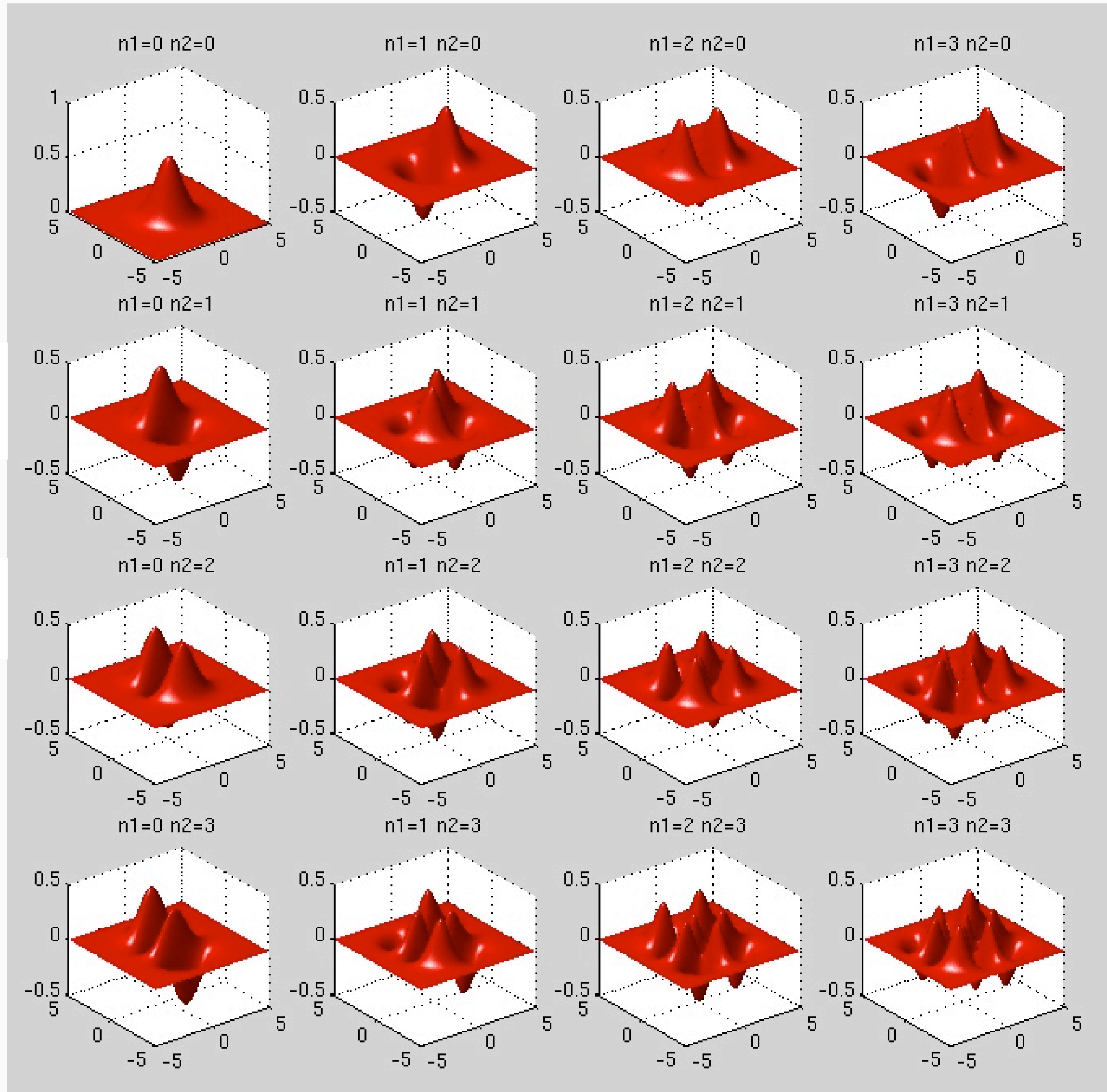
Shapelets: 2-Dimensional Basis Functions

One dimensional basis function:

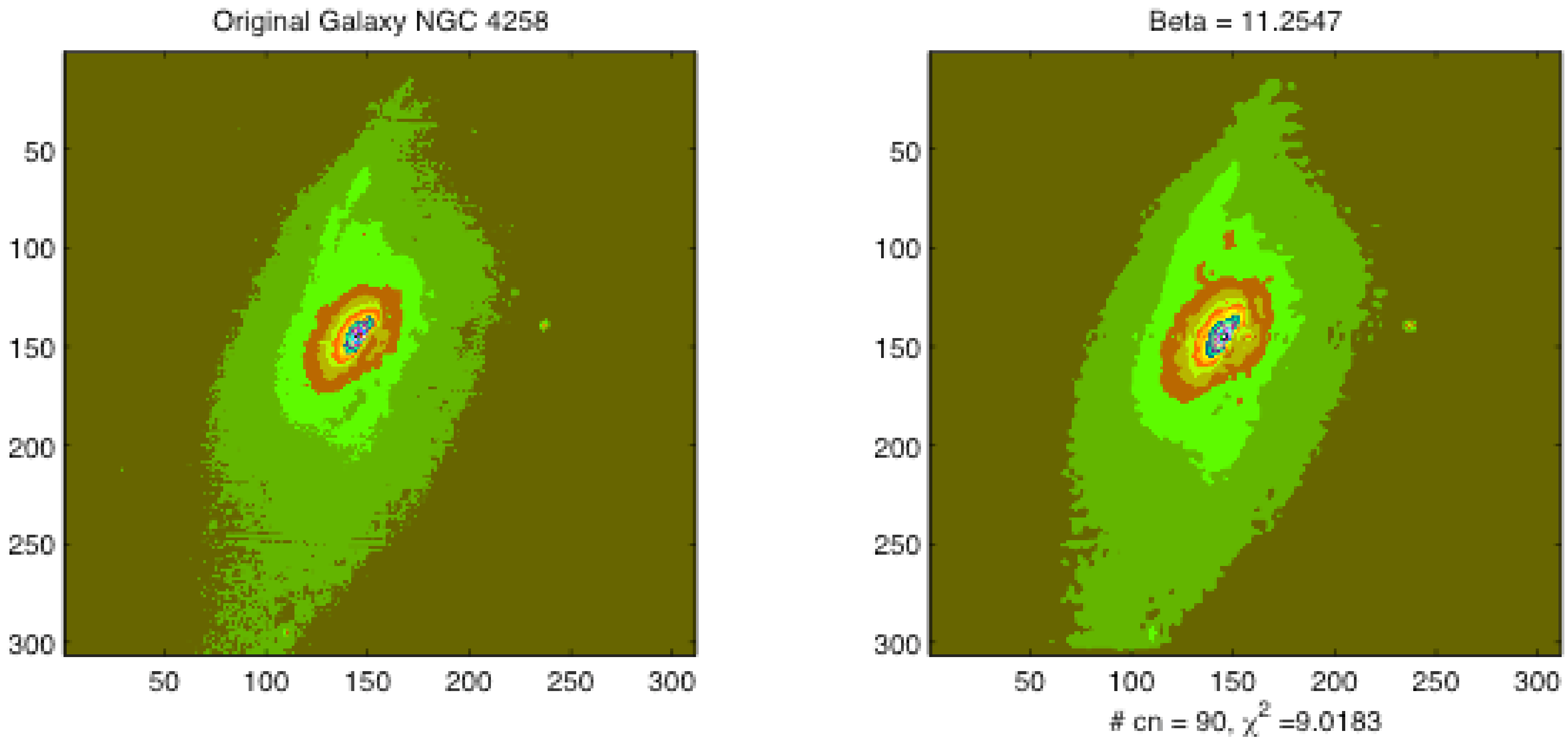
$$B_n(x;\beta) \equiv [2^n \pi^{\frac{1}{2}} n! \beta]^{-\frac{1}{2}} H_n(\tfrac{x}{\beta}) e^{-\frac{x^2}{2\beta^2}}$$

Two dimensional, orthonormal set:

$$\int d^2x B_n(x;\beta) B_m(x;\beta) = \delta_{n1m1}\delta_{n2m2}$$



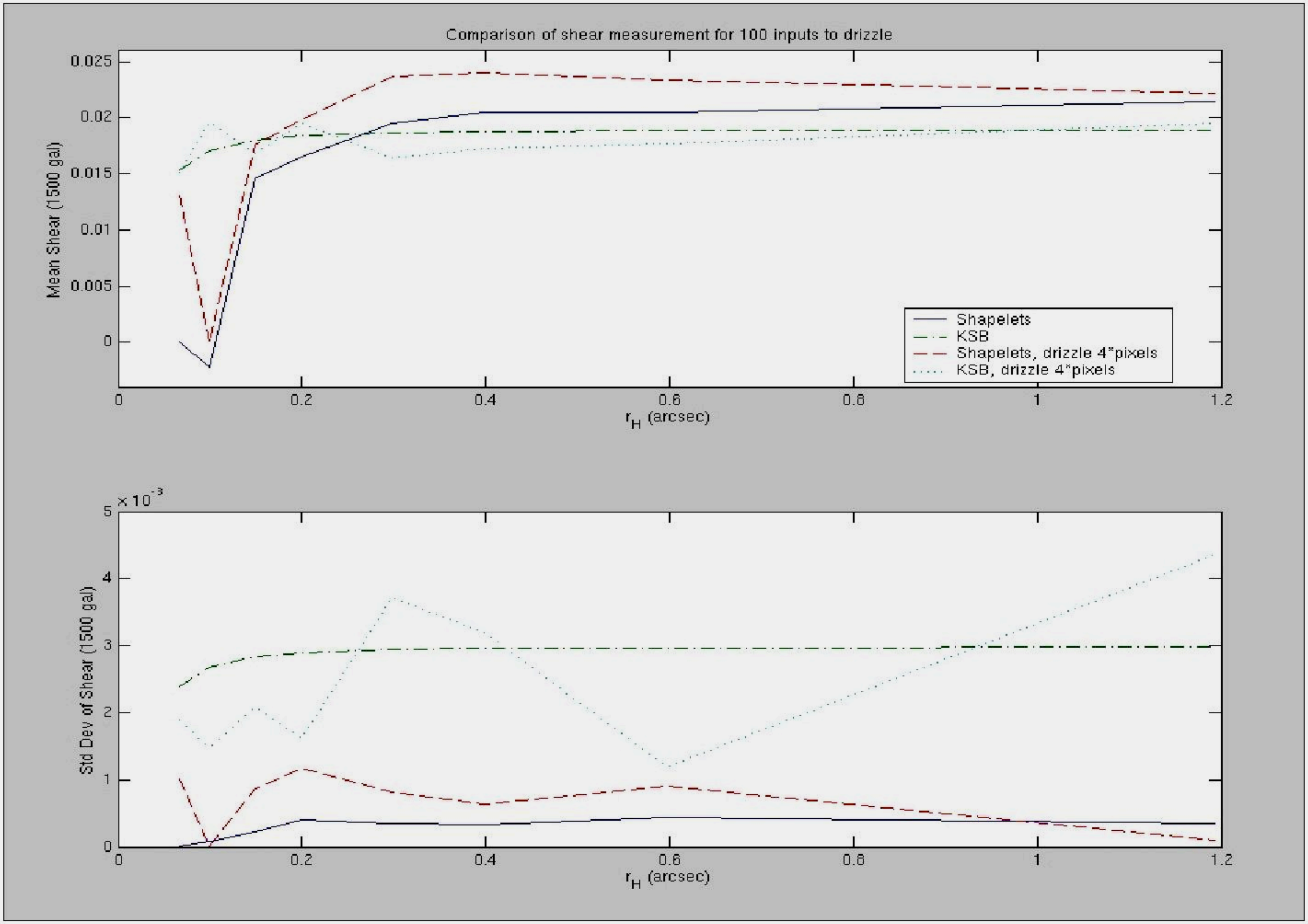
Shapelets: Deconstructed and Reconstructed Galaxy



Comparison of The Methods

	PURPOSE	MEANS	SHEAR
KSB	To use the quadrupole moments to determine the ellipticity statistic in order to correct for instrumental PSF and estimate the weak lensing shear	Uses weighted quadrupole moments to measure the ellipticity, removes the effect of the PSF anisotropy and then computes the shear	$\gamma_i \approx \epsilon_i/2$
RRG	To use the source multipole moments to correct for PSF and camera distortions, to measure the shapes of galaxies, and to estimate the weak lensing shear	Measures 0th, 2nd and 4th order moments using weight function, then corrects for anisotropic PSF and distortions resulting in corrected galaxy moments from which ellipticity and shear are computed	$\gamma_i = G^{-1}\langle\epsilon_i\rangle + O(\phi^2)$
Shapelets	To obtain reliable weak shear measurements by extracting weighted shape components, and using those weights to provide estimates of the local shear	Uses weighted Hermite polynomials to decompose 2-dimensional galaxy images. The weights, or coefficients, of these functions then provide the necessary information to measure the shear	$\tilde{\gamma}_{1n} = \frac{f_n^{'} - \langle f_n\rangle}{S_{1nm}\langle f_m\rangle}$ $\tilde{\gamma}_{2n} = \frac{f_n^{'} - \langle f_n\rangle}{S_{2nm}\langle f_m\rangle}$

KSB vs. Shapelets



Conclusions

- Accurately removing the PSF is a difficult (weighty) process
- The three methods must be tested to determine the optimal shear estimator
  - The mathematical ease of the KSB method is enticing
  - The promise of the RRG and/or Shapelets method to remove distortions and convolutions thereby reducing errors is intriguing